TOPICS IN GEOMETRIC MECHANICS: WEEK 13 TUTORIAL

LEO BUTLER

A. Let $\psi : T^*\mathbf{S}^{n-1} \to \mathfrak{so}(n)^* \equiv \mathfrak{so}(n)$ be the momentum of the standard action of $\mathrm{SO}(n)$ on $\mathbf{S}^{n-1}$.

(a) Compute the orbit of the point $\psi(P)$, where $P \in T^*\mathbf{S}^{n-1}$.

(b) Compute $\psi^* h$ where $h : \mathfrak{so}(n) \to \mathbf{R}$ is the function $h(x) = -\frac{1}{2}\mathrm{Tr}(x^2)$, $x \in \mathfrak{so}(n)$.

(c) Show that if $f \in C^\infty(\mathfrak{so}(n))$, then $\psi^* f$ Poisson commutes with $\psi^* h$.

B. Compute the Poisson structure on the dual of the Lie algebra of the Heisenberg group.

Date: April 13, 2012.