## MTH-696A: TOPICS IN GEOMETRIC MECHANICS ASSIGNMENT 7

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**A.** Let *M* be a smooth manifold and  $L: TM \to \mathbf{R}$  be a smooth, strictly convex Lagrangian. Show that (a) the energy

$$E(x,v) = \langle v, \frac{\partial L}{\partial v} \rangle - L(x,v)$$

is constant along solutions to the Euler-Lagrange equations.

- (b) the energy E equals the pull-back of H, the Hamiltonian, under the Legendre transformation.
- **B.** Compute Hamilton's equations for the spherical pendulum, using spherical coordinates  $(\theta, \varphi)$  on  $\mathbf{S}^2$  and their conjugate momenta  $(p_{\theta}, p_{\varphi})$  on  $T^*\mathbf{S}^2$ .



FIGURE 1. The spherical pendulum.

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