MTH-696A: TOPICS IN GEOMETRIC MECHANICS ASSIGNMENT 6

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A. Let $L : T\mathbf{T}^2 \to \mathbf{R}$ be the kinetic energy of the double pendulum (see assignment 3).

(a) Compute the Euler-Lagrange equations

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i} = \frac{\partial L}{\partial x_i}, \qquad x_1 = x, x_2 = y,$$

for this Lagrangian.

(b) Introduce the coordinates $x = r, y = r + s \pmod 2)\pi$. Show that

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{r}} = 0.$$

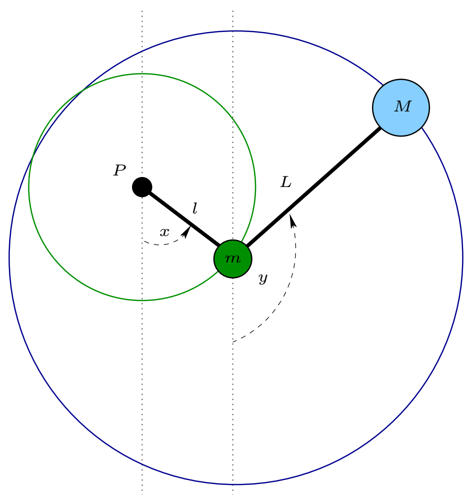

FIGURE 1. Planar double pendulum.

B. Let G be a Lie group and $\mathfrak{g}$ be its Lie algebra. Suppose that G acts linearly on $\mathbf{R}^n$; suppose also that G leaves invariant the open subset M. Let $L : TM \to \mathbf{R}$ be a smooth Lagrangian function that is constant on the G orbits:

$$L(gx, g\dot{x}) = L(x, \dot{x}), \qquad \forall g \in \mathrm{G}, (x, \dot{x}) \in TM.$$

For each $\xi \in \mathfrak{g}$, define

$$f_\xi(x, \dot{x}) = \langle \frac{\partial L}{\partial \dot{x}}, \xi x \rangle.$$

Use the Euler-Lagrange equations for L to show that

$$\frac{d}{dt} f_\xi = 0.$$

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C. Let $\mathrm{G} = \mathrm{O}(3)$, $M = \mathbf{R}^3 - \{0\}$ and

$$L = \frac{1}{2}|\dot{x}|^2 - \frac{1}{|x|}.$$

Show that the functions f_ξ you obtain can be regarded as the components of angular momentum.