MTH-696A: TOPICS IN GEOMETRIC MECHANICS ASSIGNMENT 5

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A. Let N be the Lie group

$$N = \left\{ g = \begin{bmatrix} 1 & x & z \\ 0 & 1 & y \\ 0 & 0 & 1 \end{bmatrix} \text{ s.t. } x, y, z \in \mathbf{R} \right\}$$

with the standard matrix multiplication. Let $\mathfrak{n} = T_1 N$ be the Lie algebra of N.

- (a) Compute the exponential map $\exp: T_1 N \to N$.
- (b) Show that the exponential map is a diffeomorphism; compute $\exp^{-1} : N \to T_1 N$.
- (c) Show that the centre of N is a 1-dimensional subgroup isomorphic to \mathbf{R} .
- (d) Show that the Lie algebra of N has a basis X, Y, Z such that [X, Y] = Z and all other brackets are 0 or obtained by skew-symmetry.
- (e) Let $g \in N$, and $\operatorname{Ad} : N \times \mathfrak{n} \to \mathfrak{n}$ be the adjoint action of N on \mathfrak{n} . Compute the matrix of Ad_g with respect to the basis X, Y, Z you found above.

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