MTH-696A: TOPICS IN GEOMETRIC MECHANICS ASSIGNMENT 1

DR. LEO BUTLER

A. Let $\times : \mathbf{R}^3 \times \mathbf{R}^3 \to \mathbf{R}^3$ be the vector (=cross) product. Define the following operator ω

$$\omega_x(u, v) := \langle x, u \times v \rangle, \tag{1}$$

for $x \in \mathbf{R}^3$ and $u, v \in T_x\mathbf{R}^3 \equiv \mathbf{R}^3$. Let e_1, e_2, e_3 be the standard basis of $\mathbf{R}^3$ and let x_i be coordinates induced by this basis (so $x = x_1 e_1 + x_2 e_2 + x_3 e_3$).

(a) Compute $\omega = \sum_{i<j} \omega_{ij}(x)\, \mathrm{d}x_i \wedge \mathrm{d}x_j$.

(b) Compute the exterior derivative of ω, $\mathrm{d}\omega$, and show that this equals $\mathrm{d}x_1 \wedge \mathrm{d}x_2 \wedge \mathrm{d}x_3$, the standard volume element on $\mathbf{R}^3$.

B. Let $\mathbf{S}^2 = \left\{ x \in \mathbf{R}^3 \,\text{s.t.}\, |x| = 1 \right\}$ be the unit sphere. Let $\eta = \omega|_{\mathbf{S}^2}$ be the restriction of the 2-form ω defined in (1). Show that η is a closed, non-degenerate 2-form.

Date: February 8, 2012.