Dynamical Systems and Orbital Mechanics,

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Celestial Mechanics

Johannes Kepler (1571-1630) discovered 3 laws of planetary motion:

1. a planet's orbit traces out an ellipse with the sun at one focus;

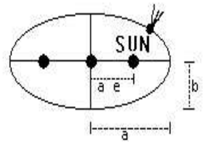

Figure:

2. equal areas are swept out in equal times;
3. the period squared of a planet's orbit is proportional to the cube of the semi-majour axis: $T^2 = kb^3$.

Celestial Mechanics

Isaac Newton (1642-1727) discovered a simple explanation:
Massive bodies exert a force on one another. The force is

1. proportional to the product of their masses;
2. inversely proportional to the square of distance of separation;
3. directed along the segment joining one another.

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Anti-intuitive Laws!

1. why should the force be proportional to the *product* of masses?
2. why inversely proportional to the *square* of distance?
3. HOW CAN BODIES ACT ON ONE ANOTHER AT A DISTANCE?

4

Celestial Mechanics

Another Question: what happens if there is a third massive body added to the system?

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Newton: a splitting headache!

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Can the equations be solved explicitly?

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A definition of **Explicit** is needed.

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Analogy with squaring the circle.

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Modern theory of Differential Galois Theory.

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Analogy with squaring the circle.

Problems:

1. perhaps our definition of explicit is too restrictive!
2. approximations...
3. geometry of solutions...

Poincaré and the Three-Body Problem

The Prize Competition.

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Four areas

1. *n*-body problem;
2. Fuchsian differential equations;
3. differential equations of Briot and Bouquet;
4. Poincaré's Fuchsian functions.

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Poincaré was awarded prize for his essay on n-body problem.

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An illustrative example

$$\sum_{k,l \in \mathbf{Z}} \frac{a_{kl}}{k\omega + l}.$$

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An illustrative example

$$\sum_{k,l \in \mathbf{Z}} \frac{a_{kl}}{k\omega + l}.$$

Convergence depends on

1. how quickly $a_{kl} \to 0$ as $k, l \to \infty$; and
2. how quickly $|k\omega + l| \to 0$.

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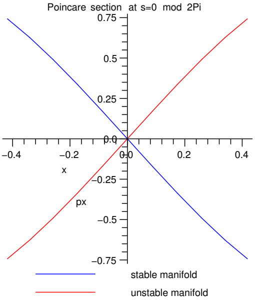

Figure: Asymptotic curves

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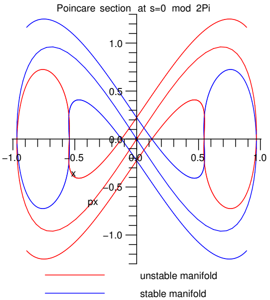

Figure: Splitting of Separatrices

Scandal

Mittag-Leffler recalls and destroys Poincaré's original manuscript.
Poincaré forced to pay for republication of new manuscript.

Legacy

Modern theory of Dynamical Systems. Topology.

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Celestial mechanics and space exploration.

Legacy

Modern theory of Dynamical Systems. Topology.

Genesis space mission.

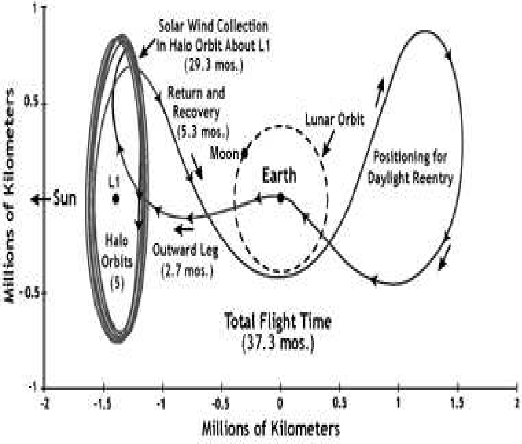

Figure: Using chaos. Source: NASA.

Suggested Reading

1. J. Barrow-Green. *Poincaré and the Three Body Problem.*
2. J. Marsden and S. Ross. New Methods in Celestial Mechanics and Mission Degign. *Bull. AMS*. 43(1), January 2006.
3. NASA's webpage *http://genesismission.jpl.nasa.gov/mission/LOI.html*.