The University of Edinburgh
2010

School of Mathematics
(U01457)

Geometry & Convergence

Problem Sheet 2

Assessment 2 due by 12.10 on Friday, 29 January 2010.
Tutorial 2 on Tuesday, 26 January 2010.

Pretutorial questions: 2, and 4.

Tutorial questions: 7, 8, and 16.

Handin question: 3.

(2**) (a) Which points in the plane $\mathbb{R}^2$ are equidistant from the x- and y-axes?

(b) Find the point that is equidistant from the x-axis, the y-axis and the line $3x + 4y = 36$.

(c) What is the largest radius of a circle that will fit inside the triangle specified by the three lines in (b)? (This circle is called the *incircle* of the triangle, and its centre is the *incentre* of the triangle).

(4**) If $\mathbf{a} \neq 0$ and $\mathbf{a} \times \mathbf{b} = 0$, prove that $\mathbf{b}$ is a scalar multiple of $\mathbf{a}$.

(7*) Solving the equation $\mathbf{a} \times \mathbf{x} = \mathbf{b}$, for given vectors $\mathbf{a}, \mathbf{b}$ in $\mathbb{R}^3$.

(a) Show that if $\mathbf{a} \times \mathbf{x} = \mathbf{b}$ then $\mathbf{a} \cdot \mathbf{b} = 0$. We assume that this condition holds for the rest of the question.

(b) Use Q4 above to show that $-(\mathbf{a}\times\mathbf{b})/|\mathbf{a}|^2$ is a solution of $\mathbf{a}\times\mathbf{x} = \mathbf{b}$.

(c) If $\mathbf{x} = \mathbf{u} - (\mathbf{a}\times\mathbf{b})/|\mathbf{a}|^2$ is another solution, show that $\mathbf{u}\times\mathbf{a} = 0$. Hence use Q3 above to write down the general solution $\mathbf{x}$ to $\mathbf{a} \times \mathbf{x} = \mathbf{b}$.

[In this question we've seen that the solution set $\{\mathbf{x} : \mathbf{a}\times\mathbf{x} = \mathbf{b}\}$ is either empty (when $\mathbf{a} \cdot \mathbf{b} \neq 0$) or a line. Note the contrast with the equation $\mathbf{a} \cdot \mathbf{x} = b$, whose solution set is a plane.]

(8*) (Converse to question 6). Show that a given line $\mathbf{w} + t\mathbf{a}$ in $\mathbb{R}^3$ is the solution set of the equation $\mathbf{a} \times \mathbf{x} = \mathbf{a} \times \mathbf{w}$.

(16*) For the plane given parametrically by $\imath(t, u) = (3, 0, 0) + t(-3, 4, 0) + u(-3, 0, 6)$ write down three points on the plane that don't lie on one line. Use these to find a normal to the plane. Find the distance of the point $(1, 2, 3)$ from the plane.