Please hand in answers no later than **Friday 26th November**.

(**Question 1**) Let $f : [0,1] \to [0,1]$ have the graph below. Prove that, for every $n \geq 1$, f has a prime period-n periodic orbit.

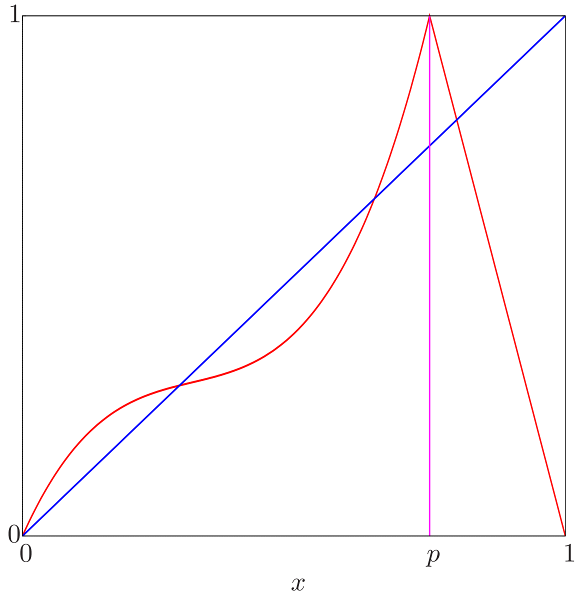

Figure .0.1: A map of $[0,1]$ with prime periodic points of all orders.

(**Question 2**) Consider the $\mathbb{R}^2$ map represented as

$$\mathbf{x}_{n+1} = \mathbf{F}(\mathbf{x}_n) \tag{0.1}$$

where

$$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}, \qquad \mathbf{F}(\mathbf{x}) = \begin{bmatrix} y + \nu \\ y^2 - x^2 \end{bmatrix},$$

and ν is a real-valued parameter.

(a) Find the fixed points of (0.1) in terms of ν.

(b) The map (0.1) undergoes a *Hopf* bifurcation when the eigenvalues of the Jacobian matrix of derivatives of $\mathbf{F}$ are $\exp(\pm i\sigma)$, where $\sigma \in (0, \pi)$. Find the value of ν, and the corresponding fixed point, at which the map undergoes a Hopf bifurcation.