Please hand in answers no later than **Friday 19th November**.

(**Question 1**) Consider the DS $x_{n+1} = F_\nu(x_n)$ where

$$F_\nu(x_n) = \nu + x^2 \,,$$

where $x \in \mathbb{R}$ and $\nu \in \mathbb{R}$.

(a) Find the fixed points. For what range of values of ν do they exist?

(b) Find the value of ν for which there is a saddle–node bifurcation.

(c) Find the value of ν for which there is a flip bifurcation. Is it super– or subcritical?

(d) Sketch the bifurcation diagram in the (ν, x) plane; indicate the stability of the fixed points in your diagram.

(**Question 2**) Consider the DS $x_{n+1} = H_\mu(x_n)$ with

$$H_\mu(x) = \mu \tan^{-1} x \,,$$

where x is a real variable and μ is a real parameter.

(a) How many fixed points are there? Specify the ranges of values of μ for which they exist.

(b) Calculate the Schwarzian derivative of H_μ.

(c) Describe the bifurcations which occur for

i. $\mu = 1$,

ii. $\mu = -1$.

If there are flip bifurcations, state whether they are supercritical or subcritical.

(d) Sketch the bifurcation diagram in the (μ, x) plane. Indicate the stability of the fixed points in your diagram.